PREDICTING PEAK OUTFLOW FROM BREACHED EMBANKMENT DAMS

Prepared for

National Dam Safety Review Board Steering Committee on Dam Breach Equations

Prepared by

M. W. Pierce, C. I. Thornton, and S. R. Abt

June 2010
Final

Colorado State University
Daryl B. Simons Building *at the*
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LIST OF SYMBOLS, UNITS OF MEASURE, AND ABBREVIATIONS

Symbols

h_d	height of the dam
h_w	height of the water behind the dam
H	height of the water behind the dam
L	dam length
Q_p	peak outflow through the dam breach
R^2	coefficient of determination
S	reservoir storage
V	volume of water behind the dam
V_w	volume of water behind the dam at failure
V_{out}	volume of outflow through the breach during failure
W_{avg}	average dam width
W_c	dam crest width
Z_{ed}	downstream embankment slope

Units of Measure

m	meter(s)
m^3	cubic meter(s)
m^3/s	cubic meter(s) per second
m^4	meter(s) to the fourth power
%	percent

Abbreviations

FEMA	Federal Emergency Management Agency
FERC	Federal Energy Regulatory Commission
ICODS	Interagency Committee on Dam Safety
M&L-M	MacDonald and Langridge-Monopolis
Reclamation	U. S. Bureau of Reclamation
SCS	Soil Conservation Service
USACE	U. S. Army Corps of Engineers
USBR	U. S. Bureau of Reclamation
USDA-ARS	U. S. Department of Agriculture - Agricultural Research Service

1 INTRODUCTION

Construction of dams has been a long-established practice with the oldest known dam, the Sadd el-Kafara near Cairo, Egypt, being built between 2950 and 2750 B.C. (Smith, 1971). Historically, many of the early dams were small, in-channel structures built by locals with little or no engineering background. Griffen (1974) suggests that dam safety for early dams was less of a concern because the areas around early dams were less densely populated and, therefore, few people were directly affected by the dams, these dams were generally small in relation to modern dams, and the dams were generally built by cultures who took pride in their work.

Modern dam-safety analysis has been an evolving science since the 1970s. Between 1972 and 1977, four notable dam failures occurred in the United States: Buffalo Creek, West Virginia; Canyon Lake, South Dakota; Teton, Idaho; and Kelly Barnes, Georgia. In April 1977, President Carter issued a memorandum directing the review of federal dam-safety activities by a committee of recognized experts. In June 1979, the Interagency Committee on Dam Safety (ICODS) issued its report containing the first dam-safety guidelines for Federal agency dam owners.

Analysis of dam breaching and the resulting floods are essential to identifying and reducing potential for loss of life and damage in the downstream floodplain. In recent years, computer modeling has become available to simulate dam-break hydrographs and route these hydrographs through the area downstream of the dam. Commonly used dam-break analysis programs require estimates of certain geometric and temporal characteristics of the dam breach as inputs for the model. Inundated areas, flow velocities, and flow depths can then be estimated to assess the potential damage caused by the dam breach as portrayed in Figure 1.

Figure 1. Artistic interpretation of the South Fork Dam breach

An alternate approach to estimating the geometric and temporal parameters of the dam breach has been the use of case-study data to develop empirical-regression relationships relating the peak discharge of the failed dam to the dam height and/or the reservoir-storage volume. Since the 1970s, multiple methodologis have been developed to estimate the peak outflow from a breached embankment dam. However, these relationships were often derived from a limited database of case studies, and confidence in these relations has been moderate.

Pierce (2008) conducted a review of the regression relationships currently utilized to estimate peak outflow from breached embankment dams. The study objectives were to: (1) review previous efforts that developed empirical relationships for estimating peak discharge from a breached embankment dam; (2) obtain new information on dam failures since 1998 and compile a database of case studies; and (3) develop enhanced relationships based on regression analysis of the case-study database.

2 BACKGROUND AND LITERATURE REVIEW

Many investigations have been conducted to develop methods used to predict the peak discharge from a breached embankment dam. Most of these investigations have used simple-regression analysis to relate the peak outflow through the breach to the depth of water behind the dam at failure, the volume of water behind the dam at failure, or the product of the depth and volume. As indicated in Table 1, the results of eleven (11) discreet investigations reported between 1977 and 1995 are presented to include the predictive expression, type of statistical curve fit, and number of case studies used in the analysis. The variables in the relationships are: Q_p = peak outflow (cubic meters per second (m^3/s)), h_w = height of the water behind the dam at failure (m), h_d = height of the dam (m), S = reservoir storage at normal pool (m^3), and V_w = volume of the water behind the dam at failure (m^3).

It is apparent that each investigator used slightly different terms to describe the effective head and volume of water that created a breach through an embankment dam. Effective head has been represented as both the height of the water behind the dam (h_w) and the height of the dam (h_d). The volume of outflow through the breach has been represented as the volume of water behind the dam at failure (V_w) and the reservoir storage (S). Additionally, definitions of reservoir storage vary for each investigator. For example, Singh and Snorrason (1984) refer to the storage term as "reservoir storage at normal pool," and Costa (1985) describes volume as the reservoir volume at the time of failure. Costa's definition of volume does not include additional inflow during a flood and presumably could include "dead storage" beneath the breach invert. Arguably, the best term to represent storage would be a measurement of the volume of outflow through the breach during failure, but in many case studies this has not been reported.

Table 1. Previous studies of peak-outflow prediction

	Investigator	Type	R^2	Number of Case Studies: Real	Number of Case Studies: Sim.	Equation and No.	
Height of Water Equations	Kirkpatrick (1977)	Best-fit	0.790[a]	13	6	$Q_p = 1.268(H_w + 0.3)^{2.5}$	**Equation 1.1**
	SCS (1981) for dams > 31.4 m	Envelope[b]	Not available	13		$Q_p = 16.6(H_w)^{1.85}$	**Equation 2.1**
	USBR (1982)	Envelope	0.724	21		$Q_p = 19.1(H_w)^{1.85}$	**Equation 3.1**
	Singh and Snorrason (1982)	Best-fit	0.488		8	$Q_p = 13.4(H_d)^{1.89}$	**Equation 4.1**
Storage Equations	Singh and Snorrason (1984)	Best-fit	0.918		8	$Q_p = 1.776\,(S)^{0.47}$	**Equation 5.1**
	Evans (1986)	Best-fit	0.836	29		$Q_p = 0.72\,(V_w)^{0.53}$	**Equation 6.1**
Height of Water and Storage Equations	Hagen (1982)	Envelope	Not available	6		$Q_p = 1.205\,(V_w \cdot H_w)^{0.48}$	**Equation 7.1**
	MacDonald and Langridge-Monopolis (1984)	Best-fit	0.788	23		$Q_p = 1.154(V_w \cdot H_w)^{0.412}$	**Equation 8.1**
	MacDonald and Langridge-Monopolis (1984)	Envelope	0.156	23		$Q_p = 3.85(V_w \cdot H_w)^{0.411}$	**Equation 9.1**
	Costa (1985)	Best-fit	0.745[c]	31		$Q_p = 0.763\,(V_w \cdot H_w)^{0.42}$	**Equation 10.1**
	Froehlich (1995)	Best-fit	0.934	22		$Q_p = 0.607\left(V_w^{\;0.295} \cdot H_w^{\;1.24}\right)$	**Equation 11.1**

[a]This R^2 value was calculated using a portion of the author's original data set.
[b]Wahl (1998) suggests that this is an enveloping equation even though 3 data points plot slightly above the curve.
[c]This R^2 value was calculated without the 5 concrete and masonry dams included in the authors original data set.

Wahl (1998, 2004) presented a database containing a composite of the case studies used by the previous investigators to develop empirical relations for predicting dam-breach parameters and peak discharge. Wahl used V_w, V_{out}, and S to represent different interpretations of the storage parameter such that V_w and V_{out} were used to report data that fit specific definitions; where V_w = volume of water stored above the breach invert at the time of failure, and V_{out} = volume of outflow through the breach during failure. The term S was used when the definition of storage was less specific.

The Wahl (1998) database was comprised of 108 case studies, forty-three (43) entries contained data describing the height (H) and volume (V) of water behind the dam at failure and an estimate of the peak outflow (Q_p) through the dam breach as presented in Table 2.

Table 2. Data collected from Wahl (1998)

	Site	Depth of Water Behind Dam at Failure (or Dam Height), H (m)	Volume of Water Behind Dam at Failure (or Storage), V (m^3)	Peak Outflow, Q_p (m^3/s)	Dam Width at Crest, W_c (m)	Average Dam Width, W_{avg} (m)	Downstream Dam Slope, Z_{ed} Z horizontal: 1 vertical	Dam Length, L (m)	Reference
1	Apishapa, CO	28	2.22E+07	6850	4.88	82.4	2		Wahl (1998)
2	Baldwin Hills, CA	12.2	9.10E+05	1130	19.2	59.6	1.8	198	Wahl (1998)
3	Break Neck Run, USA	7.0	4.90E+04	9.2		86			Wahl (1998)
4	Buffalo Creek, WV	14.02	4.84E+05	1420	128	128	1.3		Wahl (1998)
5	Butler, AZ	7.16	2.38E+06	810		9.63			Wahl (1998)
6	Castlewood, CO	21.6	6.17E+06	3570	4.9	47.4	1		Wahl (1998)
7	Davis Reservoir, CA	11.58	5.80E+07	510	6.1		2		Wahl (1998)
8	DMAD, UT	8.8	1.97E+07	793					Wahl (1998)
9	Euclides de Cunha, Brazil	58.22	1.36E+07	1020					Wahl (1998)
10	Frankfurt, Germany	8.23	3.52E+05	79					Wahl (1998)
11	Fred Burr, MT	10.2	7.50E+05	654		30.8			Wahl (1998)
12	French Landing, MI	8.53	3.87E+06	929	2.4	34.3	2.5		Wahl (1998)
13	Frenchman Creek, MT	10.8	1.60E+07	1420	6.1	37.3	2		Wahl (1998)
14	Goose Creek, SC	1.37	1.06E+07	565	3		1.5		Wahl (1998)
15	Hatchtown, UT	16.8	1.48E+07	3080	6.1	44.8	2.5	237.7	Wahl (1998)
16	Hatfield, USA	6.8	1.23E+07	3400					Wahl (1998)
17	Hell Hole, CA	35.1	3.06E+07	7360	21.3	103.2	1.5		Wahl (1998)
18	Ireland No. 5, CO	3.81	1.60E+05	110	2.4	18			Wahl (1998)
19	Johnstown, PA (South Fork)	24.6	1.89E+07	8500	3.05	64	1.5	283.5	Wahl (1998)
20	Kelly Barnes, GA	11.3	7.77E+05	680	6.1	19.4	1		Wahl (1998)
21	Lake Avalon, NM	13.7	3.15E+07	2320		42.7			Wahl (1998)
22	Lake Latonka, PA	6.25	4.09E+06	290	6.1	28			Wahl (1998)
23	Laurel Run, PA	14.1	5.55E+05	1050	6.1	40.5			Wahl (1998)

	Site	Depth of Water Behind Dam at Failure (or Dam Height), ***H*** (m)	Volume of Water Behind Dam at Failure (or Storage), ***V*** (m^3)	Peak Outflow, Q_p (m^3/s)	Dam Width at Crest, W_c (m)	Average Dam Width, W_{avg} (m)	Downstream Dam Slope, Z_{ed} ***Z*** horizontal: 1 vertical	Dam Length, ***L*** (m)	Reference
24	Lawn Lake, CO	6.71	7.98E+05	510	2.4	14.2			Wahl (1998)
25	Lily Lake, CO	3.35	9.25E+04	71					Wahl (1998)
26	Little Deer Creek, UT	22.9	1.36E+06	1330	6.1	63.1			Wahl (1998)
27	Lower Latham, CO	5.79	7.08E+06	340	4.6	25.7			Wahl (1998)
28	Lower Two Medicine, MT	11.3	2.96E+07	1800	3.7				Wahl (1998)
29	Martin Cooling Pond Dike, FL	8.53	1.36E+08	3115					Wahl (1998)
30	Mill River, MA	13.1	2.50E+06	1645					Wahl (1998)
31	Nanaksagar, IN	15.85	2.10E+08	9700					Wahl (1998)
32	North Branch, PA	5.49	2.22E+04	29.4					Wahl (1998)
33	Oros, Brazil	35.8	6.60E+08	9630	5	110			Wahl (1998)
34	Otto Run, USA	5.79	7.40E+03	60					Wahl (1998)
35	Prospect, CO	1.68	3.54E+06	116	4.3	13.1			Wahl (1998)
36	Puddingstone, CA	15.2	6.17E+05	480					Wahl (1998)
37	Quail Creek, UT	16.7	3.08E+07	3110		56.6			Wahl (1998)
38	Salles Oliveira, Brazil	38.4	7.15E+07	7200					Wahl (1998)
39	Sandy Run, PA	8.53	5.67E+04	435					Wahl (1998)
40	Schaeffer, CO	30.5	4.44E+06	4500	4.6	80.8	2	335.3	Wahl (1998)
41	South Fork Tributary, PA	1.83	3.70E+03	122					Wahl (1998)
42	Swift, MT	47.85	3.70E+07	24947				225.6	Wahl (1998)
43	Teton, ID	77.4	3.10E+08	65120	10.7	250	2.5		Wahl (1998)

2.1 Simple-regression (Single-variable) Analysis

The majority of previous investigations have used case-study data to develop empirical equations relating peak-breach discharge to the height of water behind the dam, volume of water behind the dam, or the product of the height and volume. Single-variable linear regression models were fit to case-study data to develop the following relationships.

2.1.1 Height of Water Behind the Dam (*H*)

Investigations to develop relationships relating the peak-breach outflow to the height of water behind the dam were performed by Kirkpatrick (1977), the Soil Conservation Service (SCS) (1981), the U. S. Bureau of Reclamation (USBR) (1982), and Singh and Snorrason (1982, 1984), and are listed in Table 1. Kirkpatrick (1977) analyzed data from thirteen (13) failed embankment dams and six (6) hypothetical failures. A best-fit relationship was proposed, which related the peak outflow to the depth of water behind the dam at failure. The SCS (1981) used the same thirteen (13) case studies compiled by Kirkpatrick (1977) to develop a similar relation to relate the peak outflow to the depth of water behind the dam at the time of failure. Additionally, the SCS (1981) provided a procedure for estimating peak outflow for different reservoir depths at a dam. Wahl (1998) suggested that the SCS (1981) equation was developed as an envelope relationship. Froehlich (1995) compared the SCS (1981) procedure to twenty-two (22) historical embankment-dam failures and validated the relationship for all but the smallest measured peak outflows.

To achieve consistency in defining inundated areas below USBR dams, the USBR (1982) proposed an envelope equation relating peak-breach outflow to the depth of water behind the dam. The USBR (1982) expression was developed using case-study data from twenty-one (21) failed dams including several concrete arch and gravity dams. Singh and Snorrason (1984) analyzed the results of eight (8) simulated dam failures using the U. S. Army Corps of Engineers (USACE) flood hydrograph package HEC-1 (USACE, 1978) and the National Weather Service dam-break model DAMBRK (Fread, 1979).

Figure 2 illustrates the Kirkpatrick (1977), SCS (1981), USBR (1982), and Singh and Snorrason (1984) relations, plotted with the forty-three (43) data points from Wahl (1998). It is apparent that the USBR equation provides the largest estimate of the peak outflow, while the Kirkpatrick equation represents the smallest peak-discharge estimate. The USBR, SCS, and Singh and Snorrason relationships have similar slopes and y-intercepts even though the Singh and Snorrason

(1982) equation was presented as a best-fit relationship, and the USBR and SCS equations are enveloping relationships. It is noted that none of the relationships envelop all of the data collected from Wahl (1998).

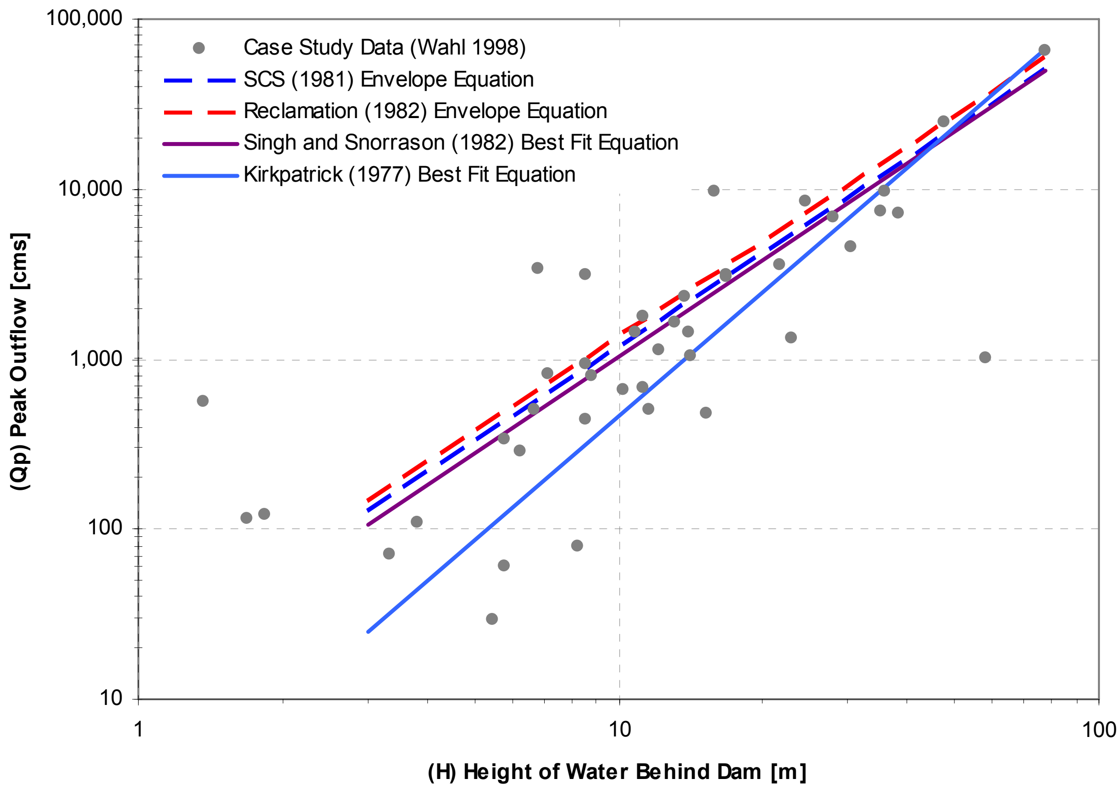

Figure 2. Peak outflow as a function of depth of water behind the dam

2.1.2 Volume of Water Behind the Dam (*V*)

Investigations to develop mathematical expressions relating the peak-breach outflow to the volume of water behind the dam at failure were performed by Singh and Snorrason (1984) as well as Evans (1986). Singh and Snorrason (1984), presented as Equation 5.1 in (Table 1) used the eight (8) simulated dam failures previously referenced and presented only the relationship relating peak outflow and volume of water behind the dam, as it exhibited the lowest standard error. To evaluate the applicability of peak-outflow relationships as a function of reservoir volume, Evans (1986) examined man-made dam failures, natural dam failures, and previous studies of jökulhlaups (glacial lake outburst floods). His investigation resulted in a relationship describing the peak outflow as a function of the outburst volume.

Figure 3 illustrates the Singh and Snorrason (1984) and Evans (1986) best-fit relations, plotted with the forty-three (43) data points from Wahl (1998). When compared to these forty-three case studies, both relationships are conservative, have similar slopes, and plot above approximately two-thirds of the data points. Over the range of these forty-three (43) case studies, peak-outflow predictions from both expressions have an average percent difference of 13%.

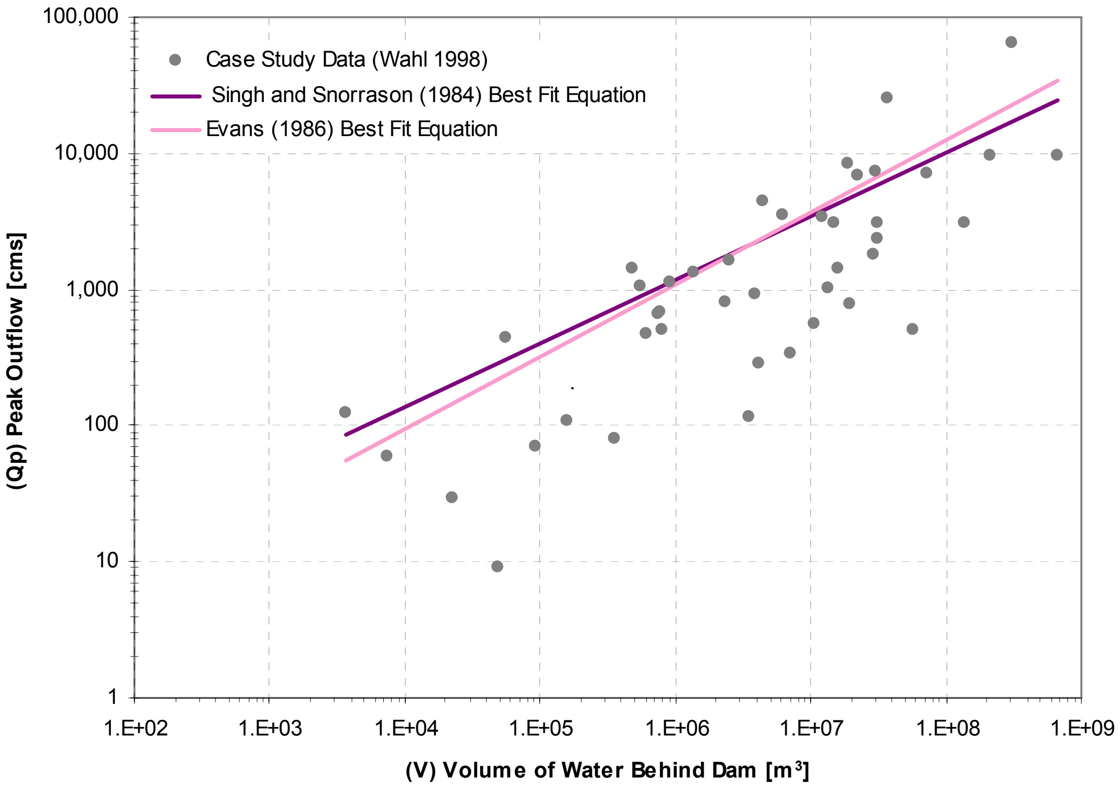

Figure 3. Peak outflow as a function of volume of water behind the dam

2.1.3 Dam Factor ($V{\cdot}H$)

In 1982, Hagen analyzed six (6) case studies of dam failures in the United States and proposed a relation which related the "dam factor" to the peak-breach outflow. Hagen (1982) defines the "dam factor" as the product of the height of the water behind the dam (H) and the reservoir storage volume at the time of failure (V). Other investigations to develop equations relating the "dam factor" to the peak-breach outflow were performed by MacDonald and Langridge-Monopolis (1984) and Costa (1985).

MacDonald and Langridge-Monopolis (1984) analyzed forty-two (42) case studies, twenty-three (23) of which included information regarding peak outflow. These 23 case studies were used to develop best-fit and envelope equations for peak outflow as a function of the dam factor. Costa (1985) analyzed thirty-one (31) dam failures and presented envelope curves and best-fit relationships based on linear regression analysis of the case studies. The proposed best-fit relationship, presented as Equation 10.1 (Table 1) also predicts the peak-breach discharge as a function of the dam factor.

Figure 4 illustrates the two-envelope relationships: MacDonald and Langridge-Monopolis (M&L-M) (1984) and Hagen (1982), as well as the two best-fit relationships: Costa (1985) and MacDonald and Langridge-Monopolis (1984). These four (4) relations are plotted with the forty-three (43) data points from Wahl (1998). Both of the envelope relationships, MacDonald and Langridge-Monopolis (1984) and Hagen (1982) envelop all but one of the case studies. Figure also depicts that the Costa (1985) and MacDonald and Langridge-Monopolis (1984) best-fit equations have very similar slopes with the MacDonald and Langridge-Monopolis equation being the more conservative.

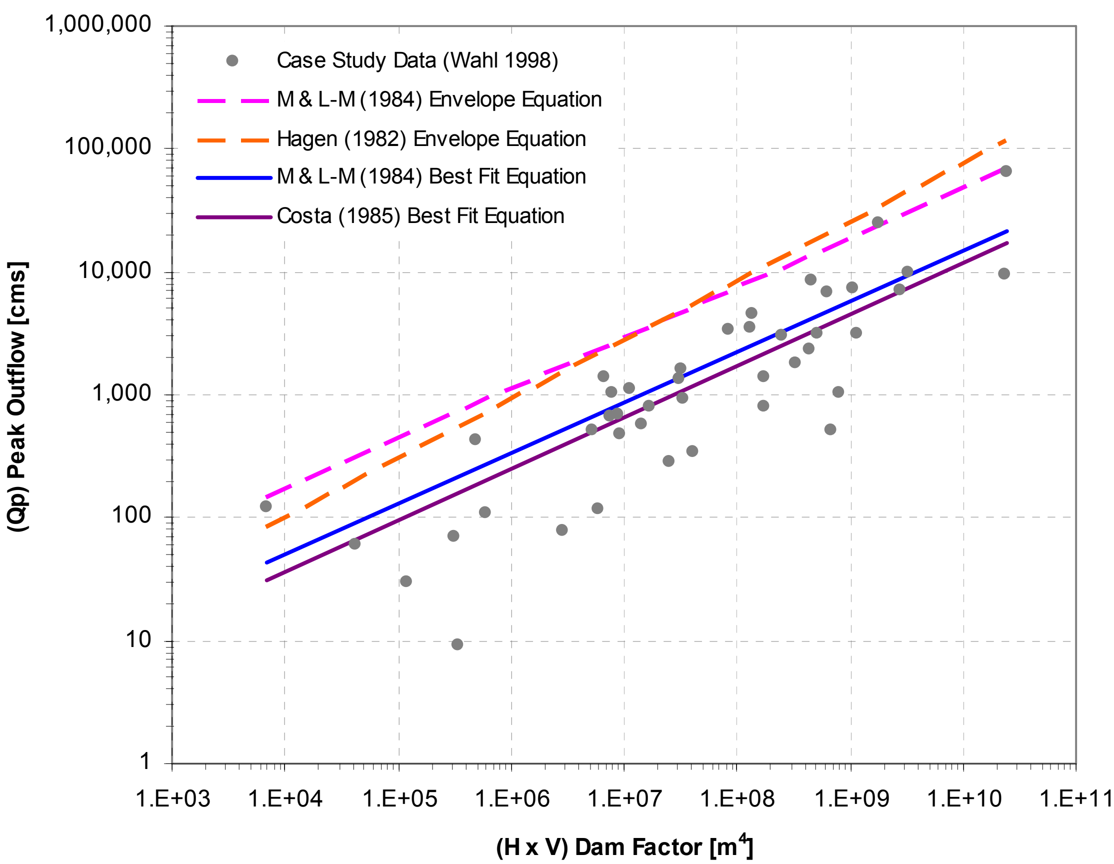

Figure 4 Peak outflow as a function of the dam factor (*V·H*)

2.2 Multiple-regression Analysis

Froehlich (1995) introduced a best-fit relationship for predicting peak outflow as a power function of both the volume and depth of water stored behind a dam. A series of twenty-two (22) case studies was analyzed using multiple-regression analysis to develop Equation 11.1 presented in Table 1. Wahl (1998) used the Froehlich (1995) relationship to predict peak outflows for thirty-two (32) case studies, including the twenty-two (22) used in the development of the equation. Based on his analysis, Wahl (1998) suggests that the Froehlich relationship is one of the better methods for direct prediction of peak-breach outflow.

Figure5 illustrates the results of using the Froehlich (1995) relationship, presented as Equation 11.1 (Table 1) to predict peak outflows for the forty-three (43) case studies from Wahl (1998). These case studies include the twenty-two (22) studies used to develop Equation 11.1 (Table 1). Several of the case studies deviate from the Froehlich relation. It is speculated that the reason for the deviations is the uncertainty surrounding the method and details of peak-flow determination for the case studies in question. The average percent difference between the observed and predicted peak outflows is approximately 119% with a maximum percent difference of 1,682%. The largest percent differences occur in case studies where the observed peak outflow is less than 1,050 m^3/s. For case studies where the observed peak outflow is greater than 1,050 m^3/s, the percent difference ranges from 4% to 113% with an average percent difference of 34%.

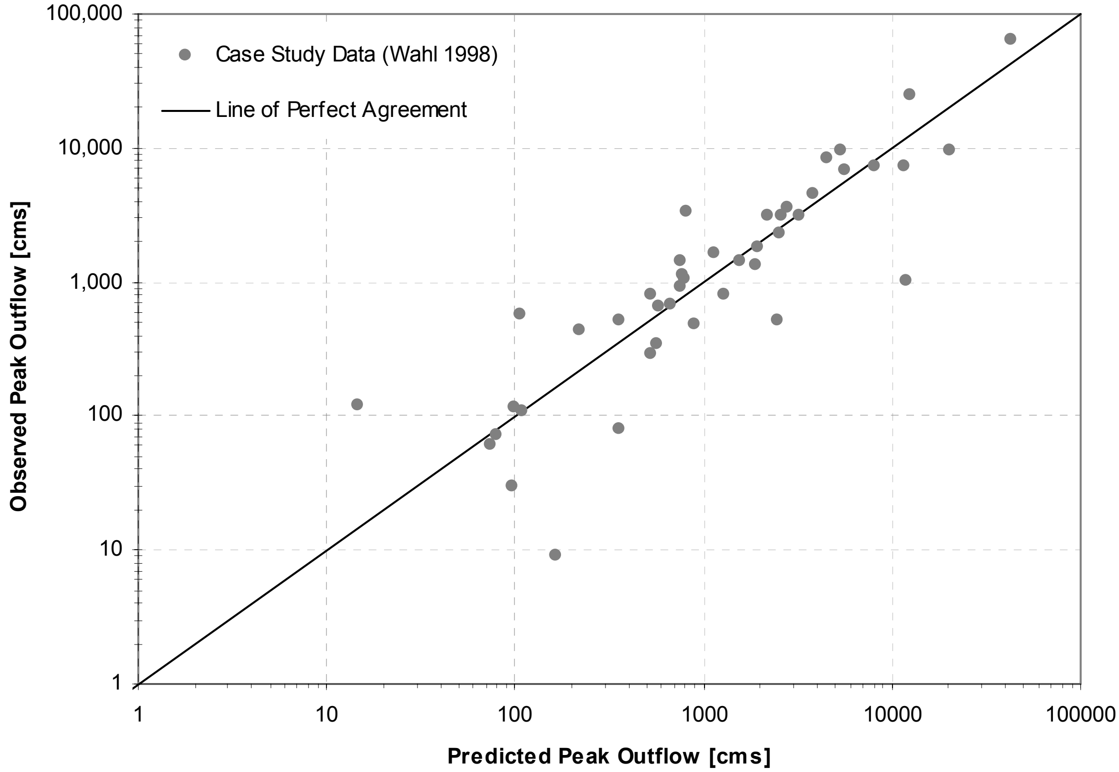

Figure 5. Observed and predicted peak discharges using the Froehlich (1995) relationship

3 EXPANDING THE DATABASE

The peak-discharge relations presented have been based on data from thirty-one (31) or fewer case studies. Since the development of these relationships, several dams have failed providing additional case study information. Also, large- and small-scale laboratory research has been undertaken to improve the understanding of embankment breaching mechanisms and processes; and provide additional data for numerical model development, calibration, and validation.

Pierce (2008) acquired dam-breach failure data from forty-four (44) case studies for breaches occurring from 1975 through 2007. Efforts to collect this information included: (1) a survey of State Dam Safety Officials from all fifty (50) states and Puerto Rico; (2) a review of available publications reporting dam failures; (3) a review of published research and testing reports; and (4) a query of the National Performance of Dams Program's dam-failure database. A summary of these embankment-dam failures is presented in Table 3. The additional data provide dam-failure information for dam heights ranging from 0.60 to 31.46 m, and peak outflows ranging from 0.28 to 78,000 m^3/sec.

Dam-breach data were collected (e.g., dam height, estimated peak outflow, water-storage volume, embankment length, etc.) from 44 additional case studies. A summary of these embankment-dam failures is presented in Table 3. The additional data provide dam-failure information for dam heights ranging from 0.60 to 31.46 m, and peak outflows ranging from 0.28 to 78,000 m^3/s.

Table 3 Case Studies Reported by Pierce (2008)

	Site	Depth of Water Behind Dam at Failure (or Dam Height), H (m)	Volume of Water Behind Dam at Failure (or Storage), V (m^3)	Peak Outflow, Q_p (m^3/s)	Dam Width at Crest, W_c (m)	Average Dam Width, W_{avg} (m)	Downstream Dam Slope, Z_{ed} Z horizontal: 1 vertical	Dam Length, L (m)	Reference
1	Banqiao, China	26.11	6.12E+08	78000				2000.00	Fujia and Yumei (1994)
2	Big Bay Dam, MS	13.59	1.75E+07	4160	12.19		3	576.07	Burge (2004)
3	Boydstown, PA	8.96	3.58E+05	65.13					SCS (1986)
4	Caney Coon Creek, OK	4.57	1.32E+06	16.99					SCS (1986)
5	Castlewood, OK	21.34	4.23E+06	3570					SCS (1986)
6	Cherokee Sandy, OK	5.18	4.44E+05	8.5					SCS (1986)
7	Colonial #4, PA	9.91	3.82E+04	14.16					SCS (1986)
8	Dam Site #8, MS	4.57	8.70E+05	48.99					SCS (1986)
9	Field Test 1-1, Norway	6.1	7.30E+04	190					Hassan et al. (2004)
10	Field Test 1-2, Norway	5.9	6.30E+04	113					Hassan et al. (2004)
11	Field Test 1-3, Norway	5.9	6.30E+04	242					Vaskinn et al. (2004)
12	Field Test 2-2, Norway	5	3.59E+04	74					Hassan et al. (2004)
13	Field Test 2-3, Norway	6	6.73E+04	174					Vaskinn et al. (2004)
14	Field Test 3-3, Norway	4.3	2.20E+04	170					Vaskinn et al. (2004)
15	Haymaker, MT	4.88	3.70E+05	26.9					SCS (1986)
16	Horse Creek #2, CO	12.5	4.80E+06	311.49					SCS (1986)
17	HR Wallingford Test 10, UK	0.6	2.45E+02	0.31					Hassan et al. (2004)
18	HR Wallingford Test 11, UK	0.6	2.45E+02	0.34					Hassan et al. (2004)
19	HR Wallingford Test 12, UK	0.6	2.45E+02	0.53					Hassan et al. (2004)
20	HR Wallingford Test 14, UK	0.6	2.45E+02	0.28					Hassan et al. (2004)
21	HR Wallingford Test 15, UK	0.6	2.45E+02	0.35					Hassan et al. (2004)
22	HR Wallingford Test 16, UK	0.6	2.45E+02	0.43					Hassan et al. (2004)
23	HR Wallingford Test 17, UK	0.6	2.45E+02	0.61					Hassan et al. (2004)

	Site	Depth of Water Behind Dam at Failure (or Dam Height), H (m)	Volume of Water Behind Dam at Failure (or Storage), V (m^3)	Peak Outflow, Q_p (m^3/s)	Dam Width at Crest, W_c (m)	Average Dam Width, W_{avg} (m)	Downstream Dam Slope, Z_{ed} Z horizontal: 1 vertical	Dam Length, L (m)	Reference
24	Lake Tanglewood, TX	16.76	4.85E+06	1351					SCS (1986)
25	Little Wewoka, OK	9.45	9.87E+05	42.48					SCS (1986)
26	Lower Reservoir, ME	9.6	6.04E+05	157.44					SCS (1986)
27	Middle Clear Boggy, OK	4.57	4.44E+05	36.81					SCS (1986)
28	Murnion, MT	4.27	3.21E+05	17.5					SCS (1986)
29	Owl Creek, OK	4.88	1.20E+05	31.15					SCS (1986)
30	Peter Green, NH	3.96	1.97E+04	4.42					SCS (1986)
31	Shimantan, China	26.55	1.11E+08	30000				500	Fujia and Yumei (1994)
32	Site Y-30-95, MS	7.47	1.42E+05	144.42					SCS (1986)
33	Site Y-36-25, MS	9.75	3.58E+04	2.12					SCS (1986)
34	Stevens Dam, MT	4.27	7.89E+04	5.92					SCS (1986)
35	Site Y-31A-5, MS	9.45	3.86E+05	36.98					SCS (1986)
36	Taum Sauk Reservoir, MO	31.46	5.39E+06	7743	3.66		1.3	2000.10	FERC (2006)
37	Upper Clear Boggy, OK	6.1	8.63E+05	70.79					SCS (1986)
38	Upper Red Rock, OK	4.57	2.47E+05	8.5					SCS (1986)
39	USDA-ARS Test #1, OK	2.29	4.90E+03	6.5			3	7.3	Hanson et al. (2005)
40	USDA-ARS Test #3, OK	2.29	4.90E+03	1.8			3	7.3	Hanson et al. (2005)
41	USDA-ARS Test #4, OK	1.5	5.09E+03	2.3			3	4.9	Hanson et al. (2005)
42	USDA-ARS Test #6, OK	1.5	5.19E+03	1.3			3	4.9	Hanson et al. (2005)
43	USDA-ARS Test #7, OK	2.13	4.77E+03	4.2			3	12	Hanson et al. (2005)
44	Wheatland Reservoir #1, WY	12.19	1.15E+07	566.34					SCS (1986)

FERC = Federal Energy Regulatory Commission

Combining the Pierce (2008) database forty-four (44) cases with the Wahl (1998) data forty-three (43) cases yields a composite dam-failure database of eighty-seven (87) case studies. A plot of peak outflows versus height of water behind the dam for the composite data is presented in Figure 6. The additional data collected during the study doubled the number of small dams (less than 10-m high) included in the composite database. Additional, the Pierce (2008) data included two case studies of dam failures where the peak outflow exceeded 30,000 m^3/s.

It is recognized that the reliability of the data presented from each case study herein is highly suspect. For example, the method and/or location of peak discharge determination in the forensics of dam failure case studies were often not reported in the documentation, thereby the reliability of the peak discharge may be within plus or minus an order of magnitude. However, including all available information in the data base as reported from the source for analysis is essential to establish the state-of-the-art for peak discharge prediction, therefore arbitrary selectivity or removal of data from such a limited base was not considered warranted.

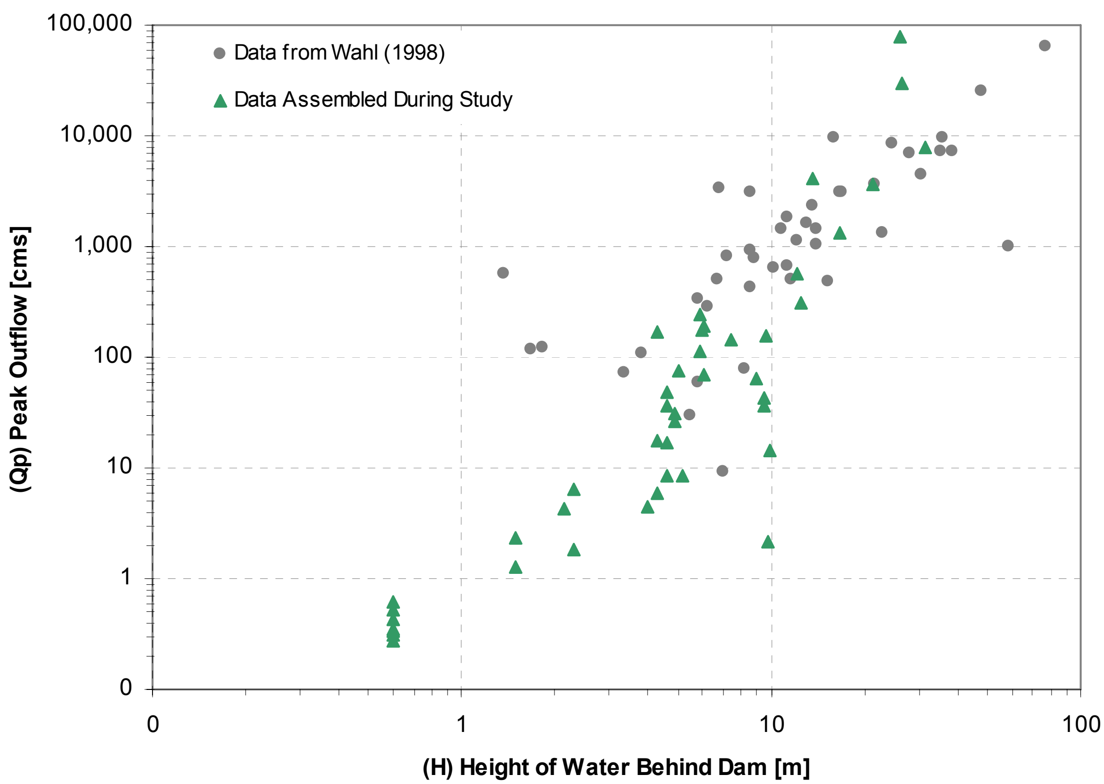

Figure 6 Plot of the 87 case studies in the composite database

4 ANALYSIS OF H, V, W AND L TERMS

A series of regression analyses was performed using the composite database. A summary of this analysis is presented. Two terms were used to represent the effective head and storage parameters. The term "H" represents the height of the water behind the dam at failure and was used to combine the h_w and h_d terms previously presented. In all case studies where the height of the water behind the dam at failure (h_w) was reported, H was used in the place of h_w. If the h_w term was not reported, and the dam failed by overtopping, the height of the dam (h_d) was used as a substitute for h_w. If the dam failed by means other than overtopping and the h_w term was not reported, the case study was not used. The term "V" represents the volume of water behind the dam at failure and was used to combine the terms V_w and S. If a value of V_w was reported, V was used in place of V_w. If the V_w term was not reported, S was assumed to be an approximation of V_w.

The Pierce et al. (2010) database contained thirty-eight (38) studies that reported dam length (L), average dam width (W_{ave}), or both length and width information (25 studies reporting average dam width, 14 studies reporting dam length, and 4 studies with both dam length and average width. A series of regression analyses will be performed to correlate these terms to peak discharge as embankment failure as well.

4.1 Linear Regression (Q_p and H)

Observation of the data presented in Figure 5 indicates that a relationship exists between the height of the water behind the dam (H) and peak outflow (Q_p). However, it is apparent that when H is less than 3 m, the data does not fit the trend of dams of greater height. Therefore, the analysis of Q_p as a function of H focused exclusively on the seventy-two (72) casse studies where H was greater than 3 m. Linear-regression analysis was performed on the logarithmic transformation of the composite data to develop a best-fit expression for predicting peak outflow from a breached embankment dam. The best-fit relation is expressed by Equation 1 and illustrated in Figure 7. The coefficient of determination (R^2) of Equation 1 is 0.675. When compared to the R^2 values of the previous relationships listed in Table 1, Equation 1 ranks in the lower 30%. However, Equation 1 was developed from an expanded database with considerable scatter:

$$Q_p = 0.784\,(H)^{2.668} \qquad \textbf{Equation 1}$$

A 95% prediction interval was developed from the composite database and can be used to evaluate the uncertainty of the scatter about the best-fit regression line. The upper boundary of this interval is expressed by Equation 2. Additionally, Froehlich (1995) provides guidance for the development of prediction limits for other exceedance probabilities.

$$Q_p = 14.68\,(H)^{2685} \qquad \textbf{Equation 2}$$

Figure 7 presents a comparison of the Pierce (2008) best-fit equation, the upper bound of the Pierce (2008) 95% prediction interval, and the relationship developed by the USBR (1982)(included for comparison). As illustrated in Figure 7, the USBR relationship is the most conservative of the historical equations used to predict peak outflow as a function of the depth of water behind the dam at failure. It is evident that approximately 90% of the additional data included in the expanded database fall below the USBR curve.

Equation 2 envelops all but one outlying data point of the composite database, as illustrated in Figure 7. Peak outflow predictions made using Equation 2 are, on average, approximately 2,100 % higher than peak-outflow predictions made using the best-fit relation described by Equation 1. For example, Banqiao Dam in China failed by overtopping in 1975. The depth of water behind the dam at failure was recorded as twenty-six (26) m and the peak outflow as approximately 78,000 m^3/s (Fujia and Yumei, 1994). The Pierce (2008) best-fit equation predicts a peak outflow of 4672 m^3/sec and the upper bound of the Pierce 95 % prediction interval of 92,455 m^3/sec, a percent difference of 1900 %.

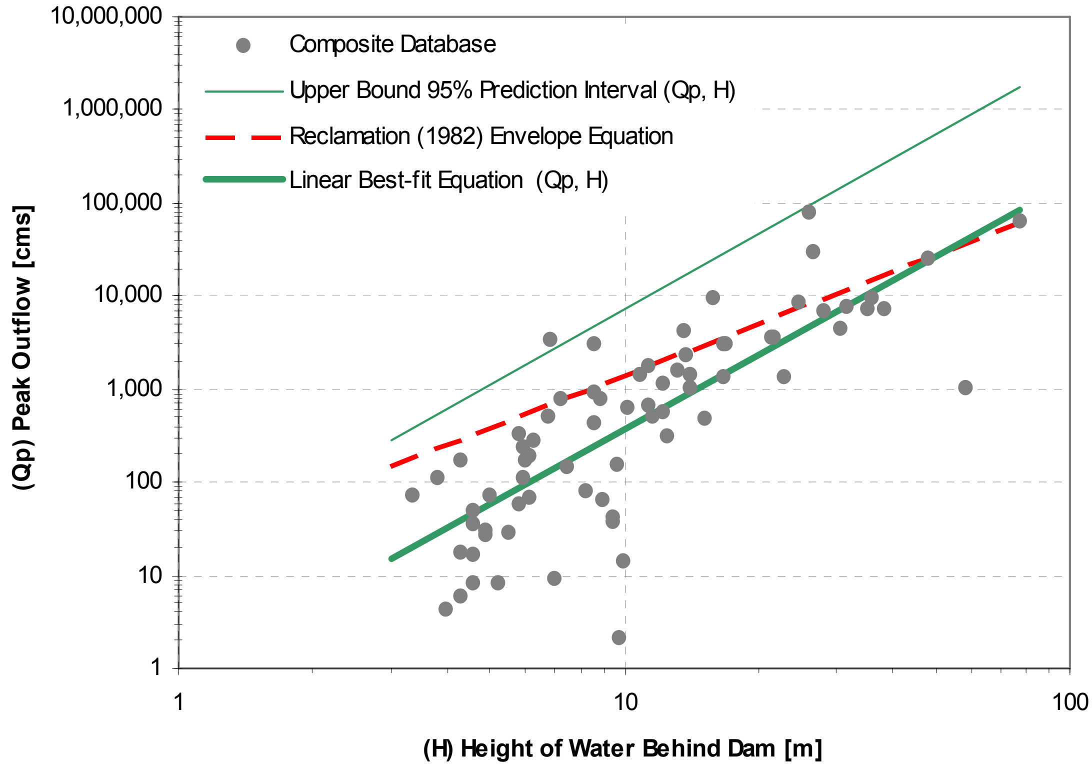

Figure 7 Comparison of the Equation 2 95% prediction interval, Equation 1 best-fit, and USBR (1982) envelope relationships

The addition of the expanded Pierce database (primarily smaller dams) to the regression analysis has significantly increased the slope and decreased the y-intercept of the best-fit relation (Equation 1) when compared to the USBR (1982) envelope equation. Near a dam height of 3 m, the 95% prediction interval and the USBR equation provide similar estimates of peak outflow, but diverge as dam height increases. Equation 1 and the USBR (1982) equation converge at a dam height of approximately 50 m.

4.2 Curvilinear Regression (Q_p and H)

A curvilinear-regression analysis was performed on the composite database to develop a best-fit expression relating the peak-breach outflow to the depth of water behind the dam at failure as expressed by Equation 3 and illustrated in Figure 8. The R^2 for the curvilinear best-fit relation expressed by Equation 4 is 0.695, higher than the R^2 of 0.633 obtained by the linear-regression relation described by Equation 1. Additionally, Equation 3 reflects the varying influence that the height of the water behind the dam has on the peak-breach outflow. If the height of the water behind the dam is considered the optimal variable to be used to predict the peak outflow from a breached embankment dam, Equation 3 enhances the prediction over Equation 1:

$$Q_p = 2.325\ \ln(H)^{6.405}$$ **Equation 3**

A 95% prediction interval, or band, was developed from the composite database and the curvilinear regression line. This interval can be used to evaluate the uncertainty of the scatter about the best-fit regression line. The upper boundary of this interval, described by Equation 4, is illustrated in Figure 7:

$$Q_p = 44.514\ \ln(H)^{6.412}$$ **Equation 4**

The relationship described by Equation 4 envelops all but one outlying data point of the composite database. Peak-outflow predictions made using Equation 4 are, on average, approximately 1,870% higher than peak-outflow predictions made using the best-fit relationship described by Equation 3.

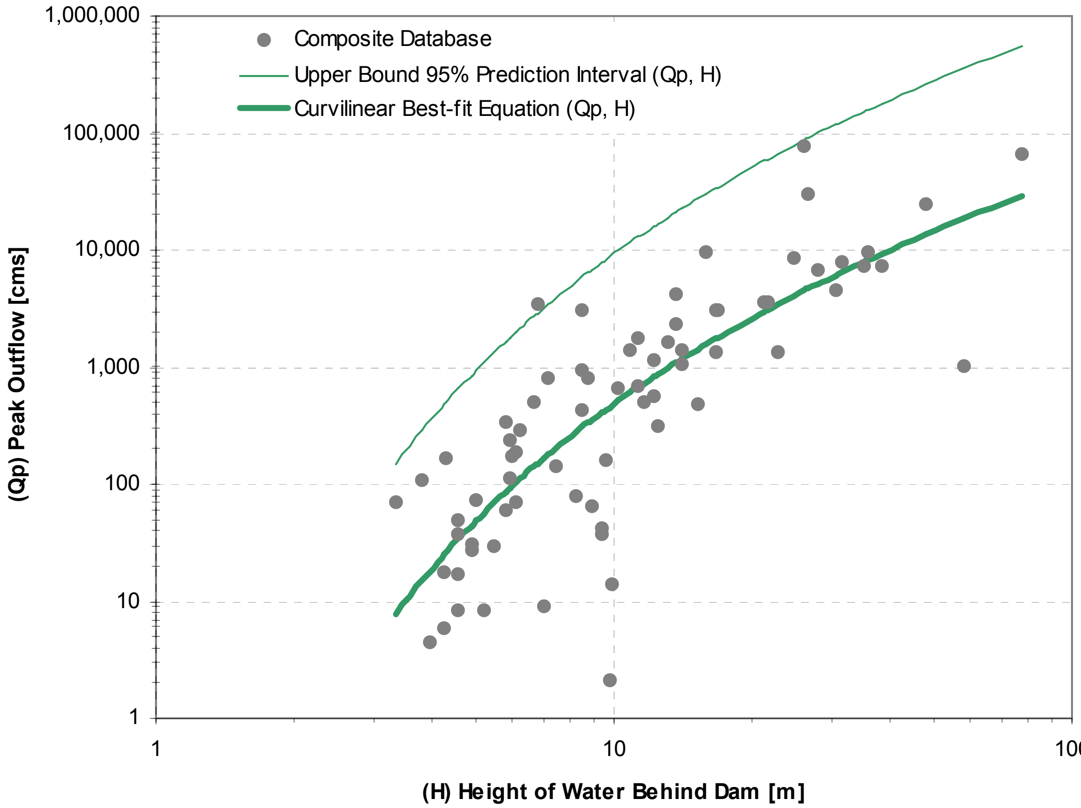

Figure 7 Curvilinear-regression analysis of the composite database ($Q_{p\ and}$ H)

4.3 Linear Regression (Q_p and V)

The volume of water behind the dam was analyzed as a predictor variable for peak-breach discharge. A linear-regression analysis was performed to determine a best-fit expression predicting peak outflow from a breached embankment dam as a function of the volume of water behind the dam. The resulting relation is described by Equation 5 and illustrated in Figure 9. The R^2 of Equation 5 is 0.805. When compared to the R^2 values of the previous relations listed in Table 1, Equation 5 ranks in the top 40%:

$$Q_p = 0.00919\,(V)^{0.745} \qquad \textbf{Equation 5}$$

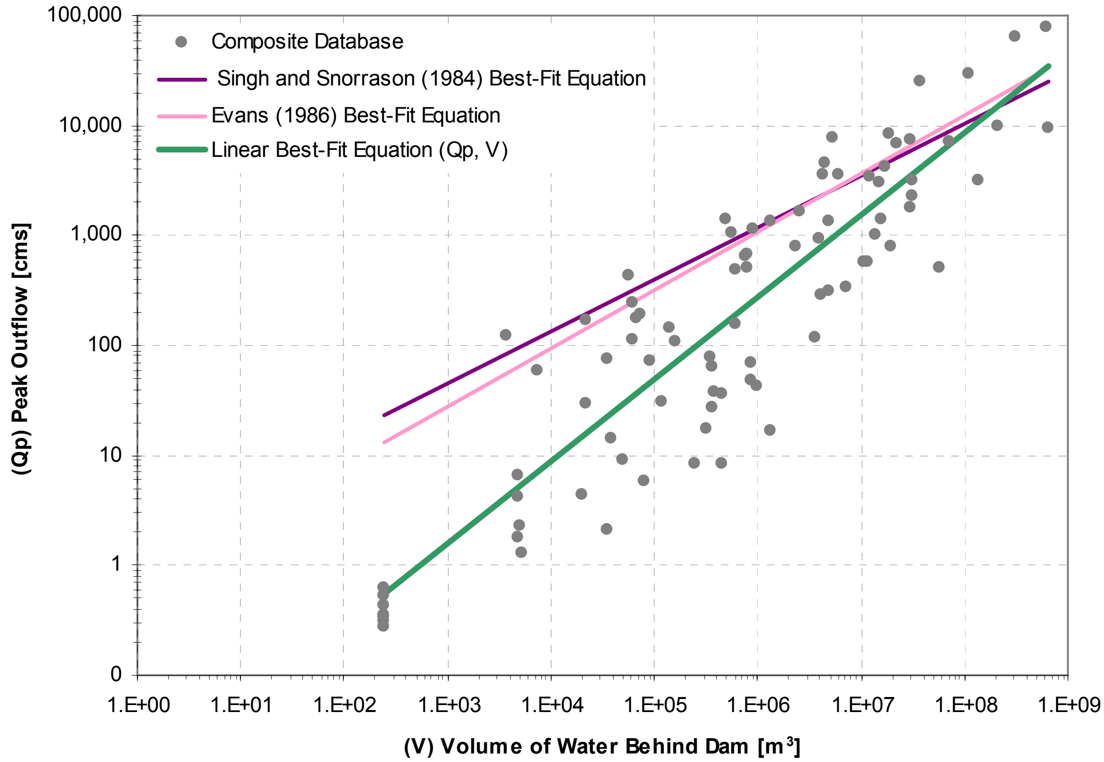

Figure 9 Comparison of the Singh and Snorrason (1984), Evans (1986), and Equation 5 linear best-fit equations

Figure 8 compares the Pierce (2008) best-fit relation developed from the regression analysis of the composite database to the Evans (1986) and Singh and Snorrason (1984) relationships. The inclusion of a larger number of low-volume reservoirs in the composite database resulted in a best-fit relation with a greater slope than historical equations. It appears that relatively small changes in the volume of water behind the dam have a greater influence on the predicted peak outflow than previously believed.

4.4 Linear Regression (Q_p and $V{\cdot}H$)

An analysis was performed with the "dam factor" expressed as the product of the height and volume of water behind the dam. Hagen (1982) used the dam factor to develop Equation 7.1 (Table 1) to predict the peak-breach outflow. Pierce (2008) performed a linear regression on the composite database using the dam factor and the peak outflow as predictor variables to develop a best-fit predictive expression. The resulting relationship is expressed by Equation 6 and illustrated in Figure 10:

$$Q_p = 0.0176\,(V \cdot H)^{0.606} \qquad \textbf{Equation 6}$$

The R^2 of Equation 6 is 0.844. When compared to the R^2 values of the previous relations listed in Table 1, Equation 6 ranks in the top 30% of all the predictive relationships. Equation 6 has a greater R^2 than other relations which use the dam factor as the dependent variable. Further, Equation 6 was developed from a database of eighty-seven (87) case studies compared to twenty-three (23) and thirty-one (31) case studies for the MacDonald and Langridge-Monopolis (1984) and Costa (1985) relationships, respectively.

Figure 10 presents a comparison of best-fit relations predicting peak outflow as a function of the dam factor, the Costa (1985), MacDonald and Langridge-Monopolis (1984), and Pierce (2008) equations. It is observed that the MacDonald and Langridge-Monopolis relationship is the more conservative of the historical equations used to predict peak outflow as a function of the dam factor. It is evident that most of the case studies with a dam factor of between 100 and 10,000,000 (m^4) plot below the MacDonald and Langridge-Monopolis curve. Thus, the best-fit relation expressed by Equation 6 has a smaller y-intercept and a steeper slope. The average percent difference between Equation 6 and the MacDonald and Langridge-Monopolis best-fit equation between 100 and 10,000,000 (m^4) is approximately 650%, while above 10,000,000 (m^4) the average percent difference reduces to approximately 40%.

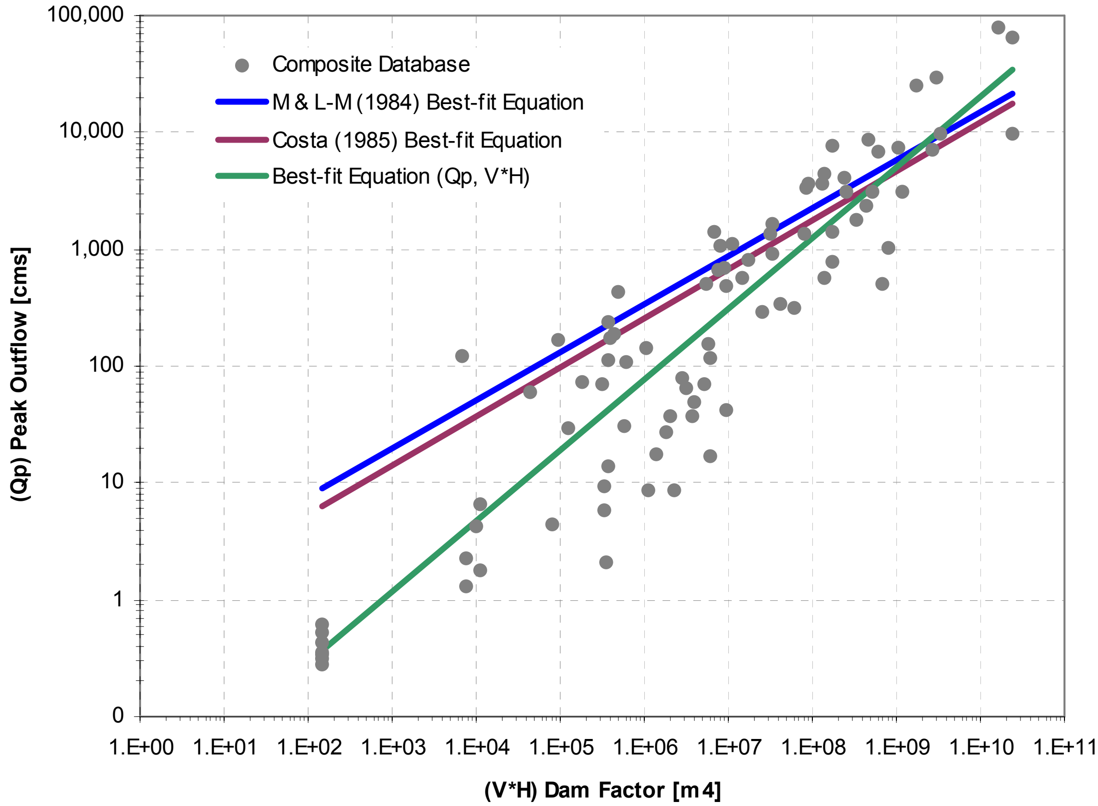

Figure 10 Comparison of the MacDonald and Langridge-Monopolis (1984), Costa (1985), and Equation 6 best-fit equations

4.5 Multiple Regression (Q_p, *V, and H*)

A multiple-regression analysis was performed on the composite database using both the height (*H*) and volume (*V*) of water behind the dam as the dependent variables. A first-order regression model was applied to the logarithmic transform of the variables and used to develop the best-fit expression described by Equation 7. The adjusted R^2 of Equation 7 is 0.850. When compared to the relations listed in Table 1, Equation 7 ranks in the top 30% and was developed from a database four times larger than Equation 1.11 (Table 1), which has an R^2 value of 0.934:

$$Q_p = 0.038\,(V^{0.475} \cdot H^{1.09}) \qquad \textbf{Equation 7}$$

Observed peak outflows are compared to the values computed by using Equation 7 in Figure 11. Equation 7 provides a reasonable agreement between observed and predicted peak outflows near the upper range of Q_p and diverges below a peak outflow of approximately 200 m^3/s. The percent error between the observed and predicted peak outflows of Equation 7 and the Froehlich (1995) relation are illustrated in Figure 12.

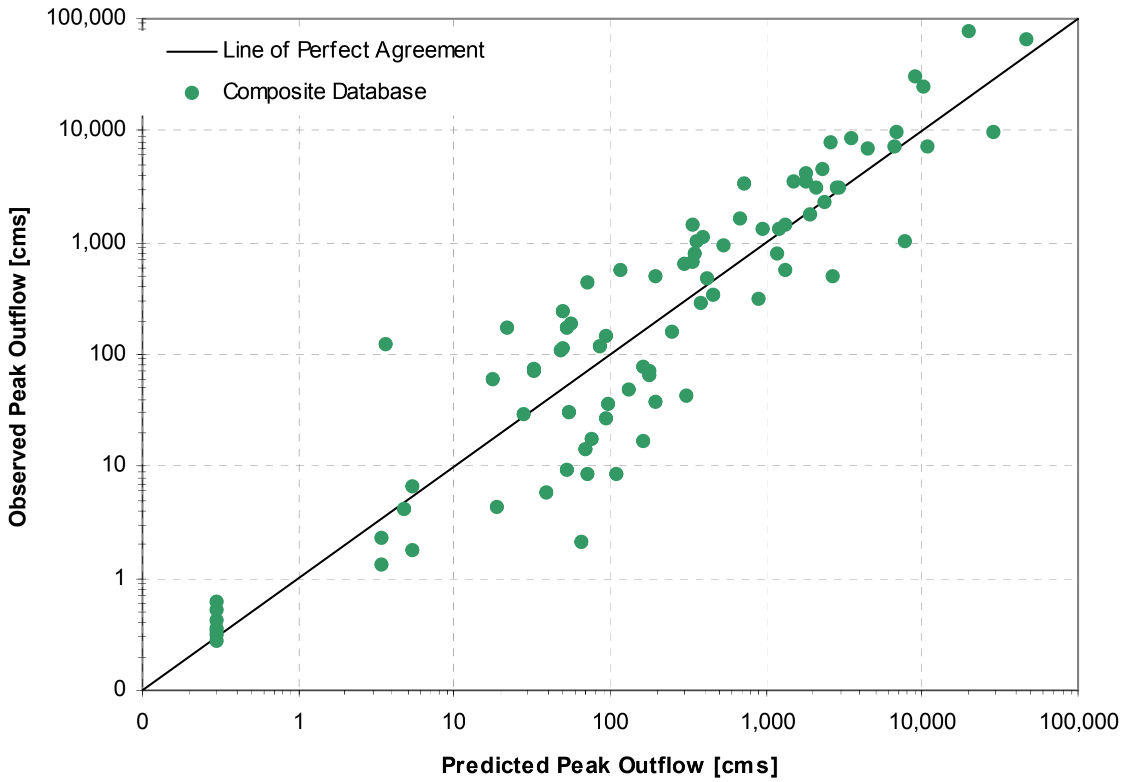

Figure 11 Observed and predicted peak discharges using the linear best-fit relationship expressed by Equation 7

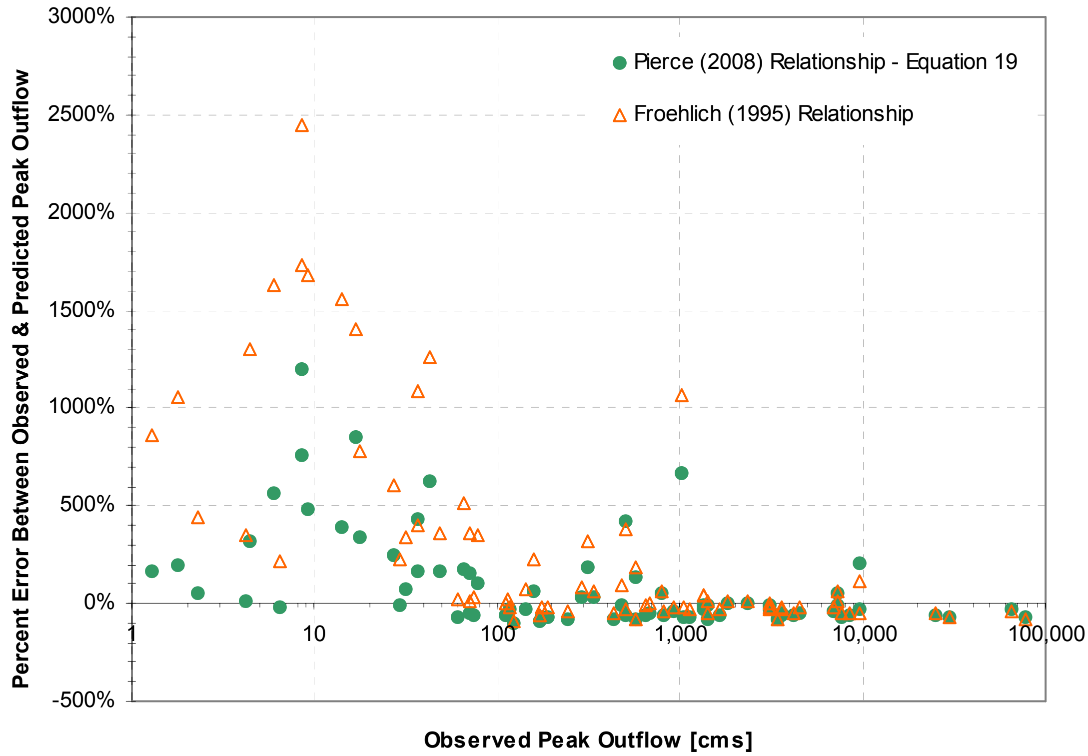

Figure 12 Comparison of percent error for Equation 7 and the Froehlich (1995) relation

Figure 12 presents a comparison of the percent error associated with using Equation 7 and the Froehlich (1995) relationship to predict peak outflow. The Froehlich (1995) expression has an average percent error of approximately 460% over the range of observed peak outflows in the composite database. Equation 7 has an average percent error of approximately 113% over the same range. This difference becomes even greater below an observed peak outflow of 200 m^3/sec. Below an observed peak outflow of 200 m3/sec, the Froehlich (1995) relation has an average percent difference of approximately 890% and Equation 7 has an average percent error of approximately 224%.

4.6 Linear Regression (Q_p and W_{avg})

The average embankment width (W_{avg}) was analyzed to determine if it could be correlated to the peak outflow at embankment failure. The composite database contains 25 case studies where both peak outflows (Q_p) and average embankment width (W_{avg}) were reported. Figure 13 presents the best-fit regression relationship for Q_p versus $W_{avg.}$ The resulting relationship has an R^2 value of 0.291 and a p-value of 0.0032. It is apparent that the average dam width is not, unto itself, a good indicator of the peak outflow.

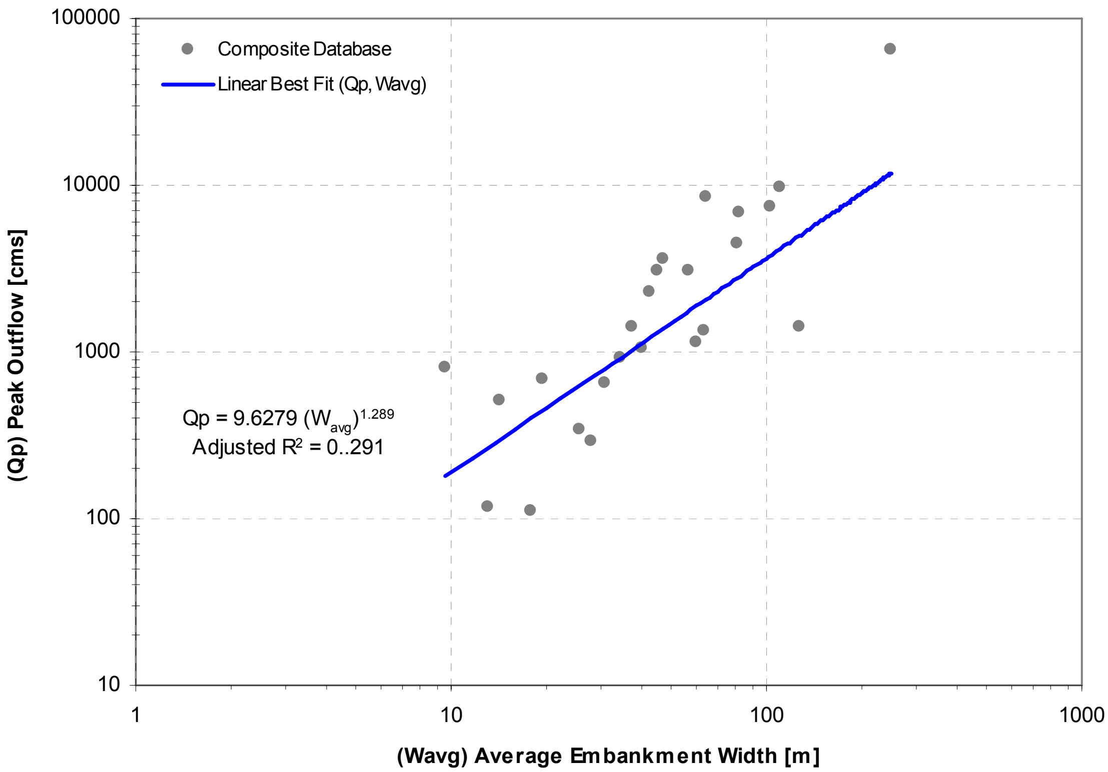

Figure 13 Analysis of average embankment width (W_{avg}) as a peak-outflow predictor

4.7 Linear Regression (Q_p and L)

Fourteen of the case studies of the composite database report both peak outflow (Q_p) and embankment dam length (L) information. A regression analysis was performed using these data resulting in a relation expressed as

$$Q_p = 0.1202\,(L)^{1.7856} \qquad \text{Equation 8}$$

The resulting relationship presented in Equation 8 has an R^2 value of 0.909 and a p-value of 8.117×10^{-8}. Figure 14 illustrates the correlation between the embankment length and the peak outflow. It is apparent that the length of the dam is a significant predictor of peak-breach outflow

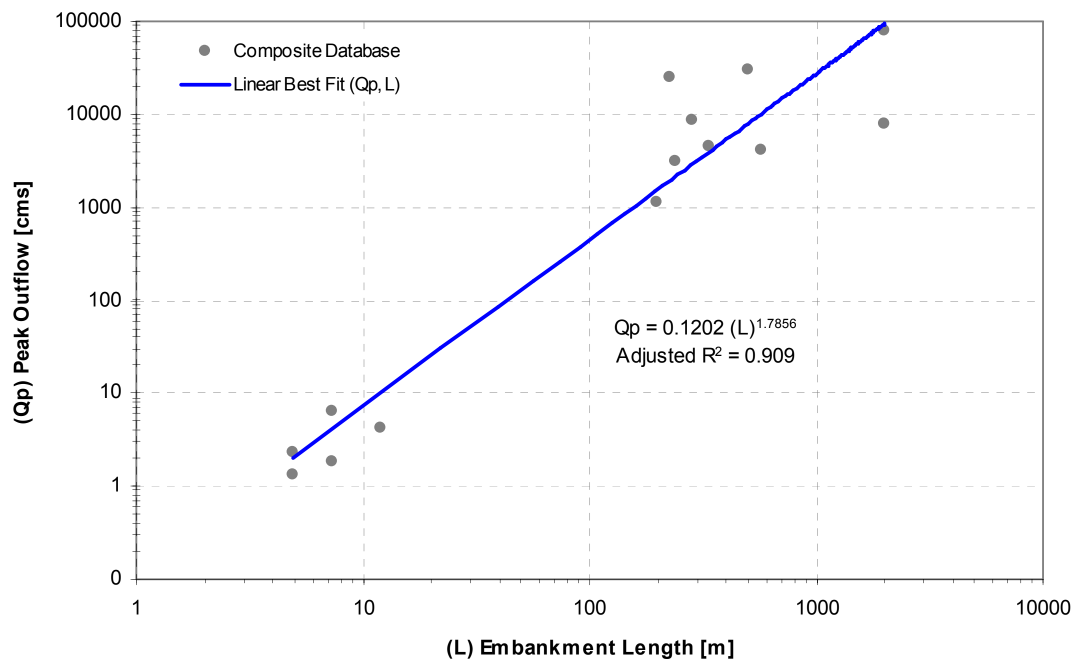

Figure 14 Analysis of embankment dam length (*L*) as a peak-outflow predictor

4.8 Multiple Regression (Q_p, V, H and W_{avg})

A multivariate regression analysis was performed using not only the volume of water behind the dam (V) and dam height (H), but also the average embankment width (W_{avg}) as dependent variables for predicting the peak outflow (Q_p) at embankment breach (25 case studies). The resulting relation for predicting the peak outflow portrayed in Figure 15 and expressed as:

$$Q_p = 0.863\left(V^{0.335} \cdot H^{1.833} \cdot W_{avg}^{-0.663}\right) \qquad \text{Equation 9}$$

Equation 9 yields an R^2 value of 0.871. The addition of the average embankment width (W_{avg}) as a dependant variable improves the prediction of the peak outflow by approximately 2.8% over using only H and V.

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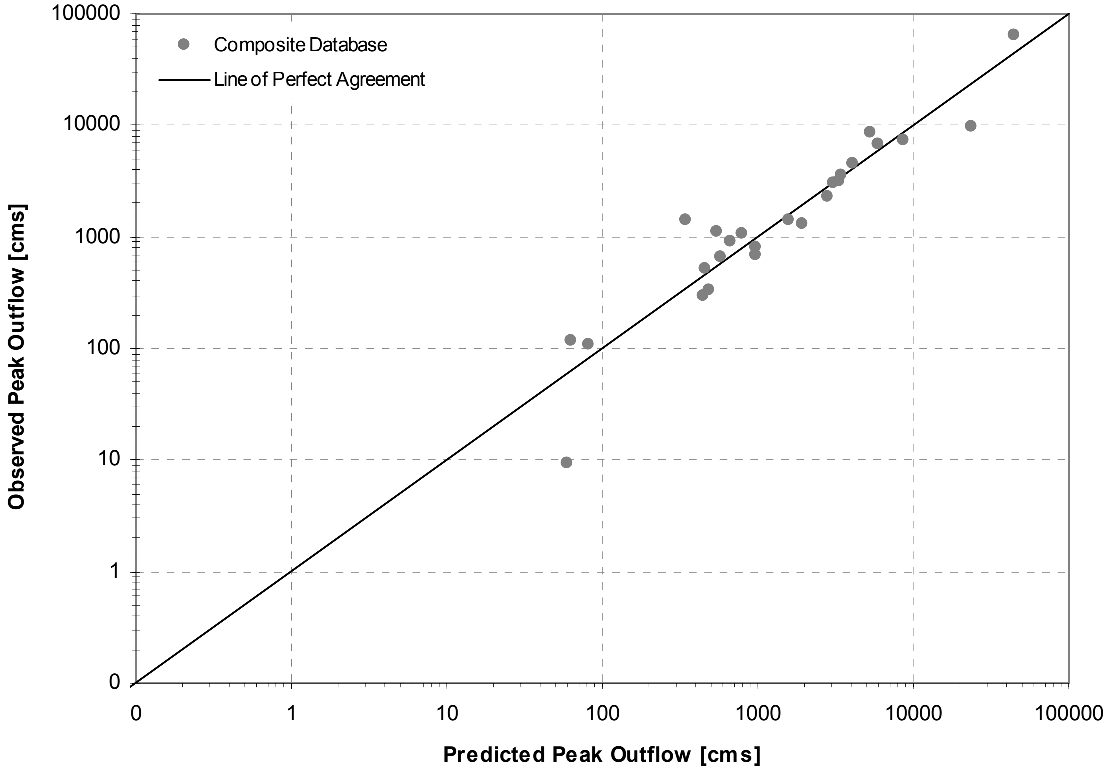

Figure 15 Observed and predicted peak discharges using the linear best-fit relationship expressed by Equation 9

4.9 Multiple Regression (Q_p, V, H and L)

A multivariate regression analysis was performed using the embankment length (L) in addition to V and H (14 case studies) similar to the W_{avg} approach presented in Equation 9. The resulting relationship for predicting the peak outflow as a function of H, V and L is portrayed in Figure 16 and expressed as

$$Q_p = 0.012\,(V^{0.493} \cdot H^{1.205} \cdot L^{0.226}) \qquad \text{Equation 10}$$

Equation 10 has an R^2 value of 0.919. Observed peak outflows are compared to the values computed applying Equation 10. The relation provides a reasonable agreement between observed and predicted peak outflows. The inclusion of L improved the correlation value by 0.25%:

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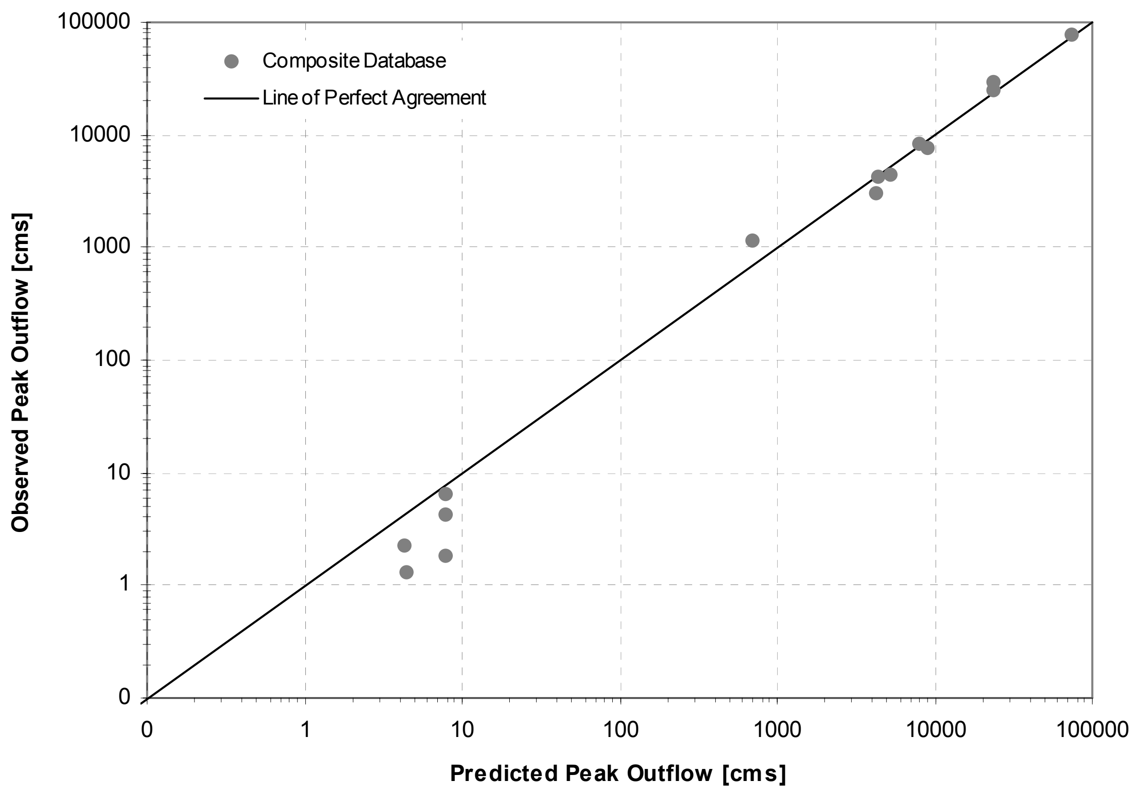

Figure 16 Observed versus predicted peak discharges using the linear best-fit relationship expressed in Equation 10.

4.10 Uncertainty Analysis

The prediction uncertainty of relations developed from statistical analysis of data collected from historic dam failures is recognized to be significant, but had never been specifically quantified until Wahl (2004). Wahl (2004) presents a description of the uncertainty analysis method used as well as a comparison of uncertainty estimates for H, V, H, V·H and (V and H) breach parameter and peak outflow prediction equations. The methods used by Wahl were applied to the Pierce (2008) equations. Table 4 presents the results of this analysis as well as a summary of the Wahl (2004) analysis.

It is observed that the Pierce relationships tend to under predict observed peak flows. On average, Equation 3 best predicts the peak outflow under estimating by only -0.058 log cycles while Equation 1 under predicts the peak outflow the most at -0.037 log cycles. The uncertainty bands for all of the presented peak outflow prediction equations range from $\pm$0.3 to $\pm$0.9 of an order of magnitude. The uncertainty bands for the Pierce equations were consistently between $\pm$0.45 and $\pm$0.60 of an order of magnitude. Equation 5 has both the lowest prediction error and the smallest uncertainty for the volume of water relations. Equation 6 has both the lowest prediction error and the smallest uncertainty for the dam factor relations. Equations 3-7 have mean prediction errors of 0.006 an order of magnitude or less.

An uncertainty analysis was also performed on the parameters of H, V and W_{avg} as well as H, V and L versus Q_p as portrayed in Table 4. It is observed that the prediction error of the relations derived from the single variable (i.e. H, V or V·H) tend to be an order of magnitude or more, greater than the relations with multiple variables (i.e. V and H; V, H and L, etc.) with the exception of the Pierce (2008) expressions. Further, peak flow prediction equations using the three variables show a slight reduction in mean predictions error (-0.005 to -0.011) over the two variable peak flow equations (-0.010 to -0.04).

Insert Table 4

A comparison of the percent error between observed and predicted peak outflows was performed for the Froehlich (1995) relation and Equation 10 as illustrated in Figure 16. It is apparent that for discharges above 1,000 cms, the Froehlich and Equation 10 relations provide similar variance, although the Froehlich relation displays a slightly higher deviation. The Froehlich relation was not developed for low flows.

The trends depicted in the prediction error analysis closely align with the comparison of band width uncertainty analyses shown in Table 4. The band width for peak flow prediction expressions using the single dependent variable ranges from approximately $\pm$0.45 to $\pm$0.93, or nearly an order of magnitude variance. The band width for the peak flow prediction using two variables reduces the variance to approximately $\pm$0.32 to $\pm$0.75, or approximately ¾ an order of magnitude. The three variable peak flow prediction relations reduce the band width uncertainty even further to $\pm$0.15 to $\pm$0.16. As the number of significant embankment characteristics increases, the band width uncertainty decreases using the case studies presented in Table 4.

A comparison of four (4) dam breach peak flow (Q_p) prediction procedures was performed to sensitize the user as to the broad range of estimates that may result applying a regression approach. From the thirty-eight (38) case studies that report forensic values for H, V, W_{avg}, L and Q_p; they include Baldwin Hills, CA., Hatchtown, UT, Johnstown, PA., and Schaeffer, CO. Two peak flow predictions procedures used were those recommended by Pierce (2008)(Eqn. 6); the $H \cdot V$ expressions derived by Froehlich

(1995)(Eqn. 11.1) and Pierce as presented in Table 5. The remaining two procedures applied in the comparison are Equation 9 (V, H and W_{avg}) and Equation 10 (V, H and L).

Insert Table 5

The case study characteristic values from Table 5 were inserted into the four (4) predictions procedures yielding the peak flows presented in Table XXX to include the reported Q_p from the dam failure forensics. It is observed that the Pierce (2008) (Eqn. 6) expression consistently underestimates the reported Q_p values by a factor of approximately 1.5 to 3. The low prediction estimates are attributed to the influence of the large number of small dams included in the composite data base. The Froehlich (1995)(Eqn. 11.1) expression also under predicts the reported Q_p values, but with a slightly improved factor of variance of approximately 1.25 to 2. Equations 9 and 10 provide similar prediction variances with predicted versus reported Q_p values differing by a factor of approximately 1.0 to 1.6. Further, it observed that Equations 9 and 10 Q_p predictions bound both above and below the reported Q_p from the case studies.

It is recognized that the four (4) case studies used for this comparison do not fully represent the spectrum of dam failures that have been recorded and therefore, the results presented in Table 5 are biased due to the incompleteness of the database. Further, Equations 9 and 10 are derived from small data pools. However, the comparison does indicate that Equations 9 and 10 portray a trend of improvement using multiple variables in performing the regression analysis, particularly as the characteristic values of the data pool expand.

4.11 COMPARISON OF RELATIONSHIPS

A comparison of selected historical and composite-data (Pierce 2008) best-fit expressions is depicted in Table 6. The historical relations in this comparison were selected for their high correlation values and non-simulated case studies. The relations are segmented into groups according to the dependent variable(s) used in the regression analysis. The number of case studies used to develop the relations is also presented.

Five relationships predicting peak outflow as a function of the height of the water behind the dam are presented in Table 5. The linear relationships, USBR (1982) expressed by Equation 3.1 (Table 1) and Pierce (2008) expressed by Equation 1 in Table 5, have similar R^2 values, 0.633 and 0.724, respectively. However, the addition of the Pierce (2008) data, primarily smaller dams, to the regression database has significantly increased the slope and decreased the y-intercept of Equation 1 when compared to the USBR (1982) envelope relationship. Figure 6 illustrates that for smaller dams, the USBR (1982) relationship is more conservative than Equation 1, although the equations converge at a dam height of approximately 50 m.

The curvilinear relation reflects the greater impact that the height of the dam has on the breach outflow for dams less than 6.5-m high, and has an R^2 value comparable to the USBR (1982) relation (0.640 and 0.724, respectively). Additionally, Equation 3 was developed from a database of seventy-seven (77) case studies compared to twenty-one (21) case studies for the USBR (1982) relation.

The Evans (1986) relationship expressed by Equation 6.1 (Table 1) and the Pierce (2008) relation (Equation 5) represent equations predicting peak outflow as a function of the volume of water behind the dam. Figure 9 illustrates that the Evans (1986) best-fit expression plots above approximately 80% of the data contained in the composite database and provides a more conservative estimate of peak outflow than Equation 5. Both relations have comparable R^2 values (0.836 and 0.805, respectively) although when plotted with the Pierce (2008) database, the Evans (1986) equation appears to be more of an enveloping relation, while Equation 5 provides a best-fit estimate of peak outflow.

The MacDonald and Langridge-Monopolis (1984) relationship expressed by Equation 8.1 (Table 1) and Pierce (2008) expressed by Equation 6 depict similar relations predicting peak-breach outflow as a function of the dam factor. Equation 6 has an R^2 value of 0.844 and the MacDonald and Langridge-Monopolis (1984) equation has an R^2 value of 0.788. In addition to an enhanced correlation, Equation 6 was developed from a database of eighty-seven (87) case studies, over three times larger than the database of twenty-three (23) case studies used to develop the MacDonald and Langridge-Monopolis (1984) relationship. Equation 6 appears to provide an improved means of predicting peak discharge using the dam factor as the dependent parameter.

Froehlich (1995) demonstrated that a multiple-regression relationship using both the height and volume of water behind the dam as regression variables can be used to predict peak outflow with reliable results. The Pierce (2008) multiple-regression relation (Equation 7) with a corresponding R^2 value of 0.850 appears to be an improved peak-discharge predictor over the Froehlich (1995) relation as Froehlich (1995) has an average percent error of approximately 460% and Equation 7 has an average percent error of approximately 113%. Below discharges of 200 m^3/s, both the Froehlich (1995) expression and Equation 7 tend to over predict peak outflows.

It is observed that the relationships developed using H as the dependent variable result in moderate correlations ranging from approximately 0.40 to 0.79. Although the height of water behind the dam (H) is the easier parameter to measure in the field, the scatter of data is significant and predictive qualities moderate. Relationships derived from case studies using the volume of water behind the dam (V) as the dependent variable display an improvement in correlation over relations using the height of the water behind the dam (H), values ranging from 0.81 to 0.84. When multiple variables are used (i.e., dam factor or multivariable), correlation values again increase ranging from 0.76 to 0.93. Based upon the analyses presented herein, the dam factor and multivariable regression approaches provide better predictive resolution than do the linear or curvilinear approaches using a single parameter, dependent variable approach. It is recognized that the composite database represents but a fraction of the number of embankment failures of record. However, until dam owners and responsible agencies improve their forensic approaches to data collection after failure, the composite database is the most comprehensive information available.

Table 6 Comparison of predictive relationships

Investigator	Number of Case	R^2	Equation and No.

		Studies			
Height of Water Behind the Dam Relationships (Q_p, H)	Linear Best-fit	72	0.633	$Q_p = 0.784\left(H\right)^{2.668}$	**Equation 1**
	Curvilinear Best-fit	72	0.640	$Q_p = 2.325\ \ln\left(H\right)^{6.405}$	**Equation 3**
	USBR (1982)[a]	21	0.724	$Q_p = 19.1\left(H_w\right)^{1.85}$	**Equation 3.1**
Volume of Water Behind the Dam Relationships (Q_p, V)	Linear Best-fit	87	0.805	$Q_p = 0.00919\ \left(V\right)^{0.745}$	**Equation 5**
	Evans (1986)	29	0.836	$Q_p = 0.72\left(V_w\right)^{0.53}$	**Equation 6.1**
Dam-factor Relationships (Q_p, $V \cdot H$)	Linear Best-fit	87	0.84	$Q_p = 0.0176\ \left(V \cdot H\right)^{0.606}$	**Equation 6**
	MacDonald and Langridge-Monopolis (1984)	23	0.788	$Q_p = 1.154\ \left(V_w \cdot H_w\right)^{0.412}$	**Equation 8.1**
Multiple Regression Relationships	Multiple Regression (Q_p, H, V)	87	0.850	$Q_p = 0.038\left(V^{0.475} \cdot H^{1.09}\right)$	**Equation 7**
	Froehlich (1995)	22	0.934	$Q_p = 0.607\left(V_w^{\ 0.295} \cdot H_w^{\ 1.24}\right)$	**Equation 11.1**
	Multiple Regression (Q_p, H, V, W_{avg})	25	0.871	$Q_p = 0.863\left(V^{0.335} \cdot H^{1.833} \cdot W_{avg}^{-0.663}\right)$	**Equation 9**
	Multiple Regression (Q_p, H, V, L)	14	0.99	$Q_p = 0.012\left(V^{0.493} \cdot H^{1.205} \cdot L^{0.226}\right)$	**Equation 10**

[a] Reclamation (1982) presents this as an envelope equation.

4.12 Case Study Comparison of Relations

A case study comparison was made using data collected from the failure of the Hatchtown Dam in Garfield County, Utah. The dam was completed in 1908 as a zoned earthfill dam approximately 237.7-m long with an average embankment width of 44.8 m. The dam failed by piping in 1914. At the time of failure, the height of the water behind the dam was approximately 16.8 m with a reservoir volume of approximately 1.48×10^7 m^3. The peak outflow through the dam breach was calculated to be 3,080 m^3/s. The peak outflow through the dam breach was predicted using selected relationships from Table 7. These relations are presented with the variables used, the predicted peak outflow, and the percent error of the predicted value.

Table 7 Comparison of predicted peak-outflow values and percent error

	Equation	Predicted Peak Outflow (m^3/s)	Percent Error	Variables	Equation No.
Height of Water Behind the Dam Relationships (Q_p, H)	Linear Best-fit	1457	-53%	Q_p, H	1
	Curvilinear Best-fit	1785	-42%	Q_p, H	3
	Reclamation (1982)[a]	3531	15%	Q_p, H	3.1
Volume of Water Behind the Dam Relationships (Q_p, V)	Linear Best-fit	2019	-34%	Q_p, V	5
	Evans (1986)	4545	48%	Q_p, V	6.1
Dam-factor Relationships (Q_p, $V{\cdot}H$)	Linear Best-fit	2154	-30%	Q_p, $H{\cdot}V$	6
	MacDonald and Langridge-Monopolis (1984)	3320	8%	Q_p, $H{\cdot}V$	8.1
Multiple Regression Relationships					
	Froehlich (1995)	2617	-15%	Q_p, H, V	11.1
	Multiple Regression (Q_p, H, V, W_{avg})	3085	0.2%	Q_p, H, V, W_{avg}	9
	Multiple Regression (Q_p, H, V, L)	4242	38%	Q_p, H, V, L	10

[a] Reclamation (1982) presents this as an envelope equation.

Percent error for the relationships using the height of water behind the dam as the primary predictor variable demonstrate a percent error ranging from 15% to -53%. Of the newly-developed relations, the curvilinear best-fit relationship (Equation 4.3) has less error than the linear best-fit (Equation 4.1), -42% and -53%, respectively. The relations predicting peak outflow as a function of the volume of water behind the dam result in a range of error from 48% for the Evans (1986) relationship to -34% for the newly-developed best-fit equation.

When multiple variables, or the product of multiple variables such as the dam factor, are used to predict peak outflow the range of percent error is reduced. The range of error for the relations using the dam factor as the primary predictor variable is the lowest of all the comparisons, from 8% to -30%. Multiple-regression relationships display a range of error from 38% for the newly-developed relationship using the length of the dam as well as the height and volume of water behind the dam as predictor variables to 0.2% for the relationship predicting peak outflow as a function of the average embankment width, the height, and the volume of water behind the dam.

5 Conclusions

From 1975 to 1995, eleven (11) historical regression relationships were developed using the height of water behind the dam (*H*), the volume of water behind the dam (*V*), the dam factor ($H{\cdot}V$), and a multivariable approach (*H and V*) to predict peak discharge (Q_p) from a breached embankment dam. These eleven (11) relationships were developed from simple- and multiple-regression analyses of a maximum of thirty-one (31) case studies.

Pierce (2008) expanded the breach database by forty-four (44) case studies yielding a composite database of eighty-seven (87) cases. Linear, curvilinear and multivariable regression analyses were performed on the composite database to develop best-fit and envelope relationships correlating the height of water behind the dam (*H*), the volume of water behind the dam (*V*), the dam factor ($H{\cdot}V$), and both the height and volume of water behind the dam (*H and V*) to the peak-breach discharge (Q_p). A comparison of selected historical and newly-derived expressions indicates that the Evans (1986), Reclamation (1982), and Froehlich (1995) relations remain valid for conservative peak-outflow predictions. The Pierce (2008) expressions using a curvilinear approach to relate Q_p as a function of *H* (Equation 3), the dam-factor analysis relating $H{\cdot}V$ and Q_p (Equation 6), and the multiple-regression relation for Q_p as a function of *H* and *V* (Equation 7) provide encouragement for practical applications where a best estimate of the peak-breach discharge is desired. When compared to historical relations, the Pierce (2008) best-fit relationship relating the *V* to Q_p (Equation 5) indicates that relatively small changes in the volume of water behind the dam have a greater influence on the predicted peak outflow than previously believed. Multivariable relationships developed using both the height and volume of water behind the dam (*H*, *V*) improve correlations over single-variable relations. Additionally, the Pierce (2008) 95% prediction intervals provide a statistical level of conservatism for peak-outflow predictions not previously developed.

Utilizing the Wahl (1998, 2004) and Pierce (2008) case study databases depicting relevant dam characteristics and peak discharge estimates at dam breach, a multivariate regression analysis was conducted. The dam characteristics of H, V, W_{avg} and L were correlated to Q_p yielding predictive relations pas presented in Equations 9 and 10. These analyses indicated that as the number of pertinent dam characteristics increase (i.e. from 1 to 3 variables), the coefficient of determination (R2) is slightly increased, the mean prediction error is reduced, and the uncertainty band width is reduced compared to previous expressions. Further, the Q_p predicted with these relations yield questionably improved results over those of Pierce (2008) and Froehlich (1995), and are far less conservative than those expressions developed prior to 1995. It is noted that these findings are based upon an extremely small pool of case study data, but not significantly smaller than the relations developed from 1977 to 1995.

It is essential for the user to understand that the regression relationships presented herein are intended as expedient approximations ($\pm$ ¼ order of magnitude) intended for predicting potential downstream damages when information and/or time is not available for a detailed analysis. Also, it is acknowledged that the quality of the data presented in these case studies may require further validation. However, these data reflect the state of the art in data collection and reporting. It is imperative that the case study data pool be expanded before confidence can be placed in using these predictive relations. The art and science of dam breach forensics, to include accessing state and federal failure files, must be improved to enhance regression prediction credibility.

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